Potential Solar Sail Degradation Effects on Trajectory and Attitude Control

Bernd Dachwald[1] and the **Solar Sail Degradation Model Working Group**[2]

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The Problem

- The optical properties of the thin metalized polymer films that are projected for solar sails are assumed to be affected by the erosive effects of the space environment
- Optical solar sail degradation (OSSD) in the real space environment is to a considerable degree indefinite (initial ground test results are controversial and relevant in-space tests have not been made so far)
- The standard optical solar sail models that are currently used for trajectory and attitude control design do not take optical degradation into account $\rightarrow$ its potential effects on trajectory and attitude control have not been investigated so far
- Optical degradation is important for high-fidelity solar sail mission analysis, because it decreases both the magnitude of the solar radiation pressure force acting on the sail and also the sail control authority
- Solar sail mission designers necessitate an OSSD model to estimate the potential effects of OSSD on their missions

Our Approach

- We established in November 2004 a "Solar Sail Degradation Model Working Group" (SSDMWG) with the aim to make the next step towards a realistic high-fidelity optical solar sail model
- We propose a simple parametric OSSD model that describes the variation of the sail film's optical coefficients with time, depending on the sail film's environmental history, i.e., the radiation dose
- The primary intention of our model is not to describe the exact behavior of specific film-coating combinations in the real space environment, but to provide a more general parametric framework for describing the general optical degradation behavior of solar sails

Overview

Different levels of simplification for the optical characteristics of a solar sail result in different models for the magnitude and direction of the SRP force acting on the sail:

Model IR (Ideal Reflection)

Most simple model

Model SNPR (Simplified Non-Perfect Reflection)

Optical properties of the solar sail are described by a single coefficient

Model NPR (Non-Perfect Reflection)

Optical properties of the solar sail are described by 3 coefficients

Generalized Model by Rios-Reyes and Scheeres

Optical properties are described by three tensors of rank 1, 2, and 3 (19 numbers in total, due to symmetry). Takes the sail shape and local optical variations into account

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SRP Force
on an Ideal Solar Sail

The solar radiation pressure (SRP) at a distance r from the sun is

$$P = \frac{S_0}{c}\left(\frac{r_0}{r}\right)^2 = 4.563\,\frac{\mu\mathrm{N}}{\mathrm{m}^2}\cdot\left(\frac{r_0}{r}\right)^2$$

$$\mathbf{F}_{\mathrm{SRP}} = 2PA\cos\alpha\cos\alpha\,\mathbf{n}$$

Nomenclature

S_0: solar constant ($1368\,\mathrm{W/m^2}$)

c: speed of light in vacuum

r_0: 1 astronomical unit (1 AU)

α: sail pitch angle

$\mathbf{n}$: sail normal vector

$\mathbf{t}$: sail tangential vector

$\mathbf{F}_{\mathrm{SRP}}$: SRP force

A: sail area

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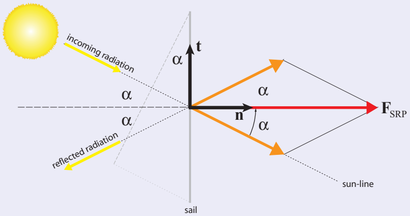

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The Non-Perfectly Reflecting Solar Sail

The non-perfectly reflecting solar sail model parameterizes the optical behavior of the sail film by the optical coefficient set

$$\mathcal{P} = \{\rho, s, \varepsilon_\mathrm{f}, \varepsilon_\mathrm{b}, B_\mathrm{f}, B_\mathrm{b}\}$$

The optical coefficients for a solar sail with a highly reflective aluminum-coated front side and with a highly emissive chromium-coated back side are:

$$\mathcal{P}_{\mathrm{Al|Cr}} = \{\rho = 0.88, s = 0.94, \varepsilon_\mathrm{f} = 0.05, \varepsilon_\mathrm{b} = 0.55, B_\mathrm{f} = 0.79, B_\mathrm{b} = 0.55\}$$

Nomenclature

ρ: reflection coefficient

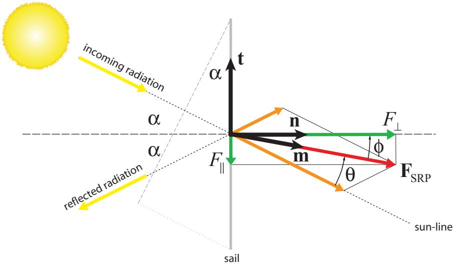
s: specular reflection factor

ε_f and ε_b: emission coefficients of the front and back side, respectively

B_f and B_b: non-Lambertian coefficients of the front and back side, respectively

The Non-Perfectly Reflecting Solar Sail

SRP Force in a sail-fixed coordinate frame $\mathcal{S} = \{\mathbf{n}, \mathbf{t}\}$



$$\mathbf{F}_{\text{SRP}} = 2PA\cos\alpha\left[\left(a_1\cos\alpha + a_2\right)\mathbf{n} - a_3\sin\alpha\,\mathbf{t}\right]$$

with the derived optical coefficients

$$a_1 \triangleq \frac{1}{2}(1 + s\rho) \qquad a_2 \triangleq \frac{1}{2}\left[B_{\text{f}}(1-s)\rho + (1-\rho)\frac{\varepsilon_{\text{f}}B_{\text{f}} - \varepsilon_{\text{b}}B_{\text{b}}}{\varepsilon_{\text{f}} + \varepsilon_{\text{b}}}\right]$$

$$a_3 \triangleq \frac{1}{2}(1 - s\rho)$$

Nomenclature

- α: sail pitch angle
- **n**: sail normal vector
- **m**: thrust unit vector
- **t**: sail tangential vector
- $\mathbf{F}_{\text{SRP}}$: SRP force
- $\mathbf{F}_{\perp}$: SRP force component along **n**
- $\mathbf{F}_{||}$: SRP force component along **t**
- θ: thrust cone angle
- ϕ: centerline angle
- P: solar radiation pressure (SRP)
- A: sail area

The Non-Perfectly Reflecting Solar Sail

SRP Force on along the radial and sail normal direction

$\mathbf{F}_{\mathrm{SRP}}$ can also be decomposed along the radial direction $\mathbf{e}_r$ and the sail normal direction $\mathbf{n}$:

$$\mathbf{F}_{\mathrm{SRP}} = 2PA\cos\alpha\left[b_1\mathbf{e}_r + (b_2\cos\alpha + b_3)\mathbf{n}\right]$$

with the derived optical coefficients

$$b_1 \triangleq \frac{1}{2}(1 - s\rho)$$

$$b_2 \triangleq s\rho$$

$$b_3 \triangleq \frac{1}{2}\left[B_{\mathrm{f}}(1 - s)\rho + (1 - \rho)\frac{\varepsilon_{\mathrm{f}} B_{\mathrm{f}} - \varepsilon_{\mathrm{b}} B_{\mathrm{b}}}{\varepsilon_{\mathrm{f}} + \varepsilon_{\mathrm{b}}}\right]$$

Nomenclature

$\mathbf{F}_{\mathrm{SRP}}$: SRP force

P: solar radiation pressure (SRP)

A: sail area

α: sail pitch angle

$\mathbf{e}_r$: radial unit vector

$\mathbf{n}$: sail normal vector

The Simplified Model

SRP Force in a sail-fixed coordinate frame $\mathcal{S} = \{\mathbf{n}, \mathbf{t}\}$

Recall that

$$\mathbf{F}_{\text{SRP}} = 2PA\cos\alpha\left[(a_1\cos\alpha + a_2)\,\mathbf{n} - a_3\sin\alpha\,\mathbf{t}\right]$$

with

$$a_1 \triangleq \frac{1}{2}(1 + s\rho)$$

$$a_2 \triangleq \frac{1}{2}\left[B_\text{f}(1 - s)\rho + (1 - \rho)\frac{\varepsilon_\text{f} B_\text{f} - \varepsilon_\text{b} B_\text{b}}{\varepsilon_\text{f} + \varepsilon_\text{b}}\right]$$

$$a_3 \triangleq \frac{1}{2}(1 - s\rho)$$

Assumptions: $s = 1$, $\varepsilon_\text{f} B_\text{f} = \varepsilon_\text{b} B_\text{b}$

$$\mathbf{F}_{\text{SRP}} = PA\cos\alpha\left[(1 + \rho)\cos\alpha\,\mathbf{n} - (1 - \rho)\sin\alpha\,\mathbf{t}\right]$$

Typically, however, the reflection coefficient ρ is denoted as η within this model

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Those models do not include **optical solar sail degradation** (OSSD)

Data Available From Ground Testing

- Much ground and space testing has been done to measure the optical degradation of metalized polymer films as second surface mirrors (metalized on the back side)
- No *systematic* testing to measure the optical degradation of candidate solar sail films (metalized on the front side) has been reported so far and preliminary test results are controversial
  - Lura et. al. measured considerable OSSD after combined irradiation with VUV, electrons, and protons
  - Edwards et. al. measured no change of the solar absorption and emission coefficients after irradiation with electrons alone
- Respective in-space tests have not been made so far
- The optical degradation behavior of solar sails in the real space environment is to a considerable degree indefinite

Simplifying Assumptions

For a *first* OSSD model, we have made the following simplifications:

1. The only source of degradation are the solar photons and particles
2. The solar photon and particle fluxes do not depend on time (average sun without solar events)
3. The optical coefficients do not depend on the sail temperature
4. The optical coefficients do not depend on the light incidence angle
5. No self-healing effects occur in the sail film

Solar radiation dose (SRD)

Let p be an arbitrary optical coefficient from the set $\mathcal{P}$. With OSSD, p becomes time-dependent, $p(t)$. With the simplifications stated before, $p(t)$ is a function of the solar radiation dose $\tilde{\Sigma}$ (dimension $\left[\mathrm{J/m^2}\right]$) accepted by the solar sail within the time interval $t - t_0$:

$$\tilde{\Sigma}(t) \triangleq \int_{t_0}^{t} S \cos\alpha \, dt' = S_0 r_0^2 \int_{t_0}^{t} \frac{\cos\alpha}{r^2} \, dt'$$

SRD per year on a surface perpendicular to the sun at 1 AU

$$\tilde{\Sigma}_0 = S_0 \cdot 1\,\mathrm{yr} = 1368\,\mathrm{W/m^2} \cdot 1\,\mathrm{yr} = 15.768\,\mathrm{TJ/m^2}$$

Dimensionless SRD

Using $\tilde{\Sigma}_0$ as a reference value, the SRD can be defined in dimensionless form:

$$\Sigma(t) = \frac{\tilde{\Sigma}(t)}{\tilde{\Sigma}_0} = \frac{r_0^2}{T} \int_{t_0}^{t} \frac{\cos\alpha}{r^2} \, dt' \quad \text{where} \quad T \triangleq 1\,\mathrm{yr}$$

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$\Sigma(t)$ depends on the solar distance history and the attitude history $\mathbf{z}[t] = (r, \alpha)[t]$ of the solar sail, $\Sigma(t) = \Sigma(\mathbf{z}[t])$

Differential form for the SRD

The equation for the SRD can also be written in differential form:

$$\dot{\Sigma} = \frac{r_0^2}{T}\frac{\cos\alpha}{r^2} \quad \text{with} \quad \Sigma(t_0) = 0$$

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Assumption that each p varies exponentially with $\Sigma(t)$

Assume that $p(t)$ varies exponentially between $p(t_0) = p_0$ and $\lim_{t\to\infty} p(t) = p_\infty$

$$p(t) = p_\infty + (p_0 - p_\infty) \cdot e^{-\lambda\Sigma(t)}$$

The degradation constant λ is related to the "half life solar radiation dose" $\hat{\Sigma}$ $(\Sigma = \hat{\Sigma} \Rightarrow p = \frac{p_0+p_\infty}{2})$ via

$$\lambda = \frac{\ln 2}{\hat{\Sigma}}$$

Note that this model has 12 free parameters additional to the 6 p_0, 6 p_∞ and 6 half life SRDs $\hat{\Sigma}_p$ (too much for a simple parametric OSSD analysis)

Reduction of the number of model parameters

We use a degradation factor d and a single half life SRD for all p, $\hat{\Sigma}_p = \hat{\Sigma}\ \forall p \in \mathcal{P}$

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EOL optical coefficients

Because the reflectivity of the sail decreases with time, the sail becomes more matt with time, and the emissivity increases with time, we use:

$$\rho_\infty = \frac{\rho_0}{1+d} \qquad s_\infty = \frac{s_0}{1+d} \qquad \varepsilon_{\mathrm{f}\infty} = (1+d)\varepsilon_{\mathrm{f}0}$$
$$\varepsilon_{\mathrm{b}\infty} = \varepsilon_{\mathrm{b}0} \qquad B_{\mathrm{f}\infty} = B_{\mathrm{f}0} \qquad B_{\mathrm{b}\infty} = B_{\mathrm{b}0}$$

Degradation of the optical parameters in dimensionless form

$$\frac{p(t)}{p_0} = \begin{cases} \left(1 + d\mathrm{e}^{-\lambda\Sigma(t)}\right) / (1+d) & \text{for} \quad p \in \{\rho, s\} \\ 1 + d\left(1 - \mathrm{e}^{-\lambda\Sigma(t)}\right) & \text{for} \quad p = \varepsilon_\mathrm{f} \\ 1 & \text{for} \quad p \in \{\varepsilon_\mathrm{b}, B_\mathrm{f}, B_\mathrm{b}\} \end{cases}$$

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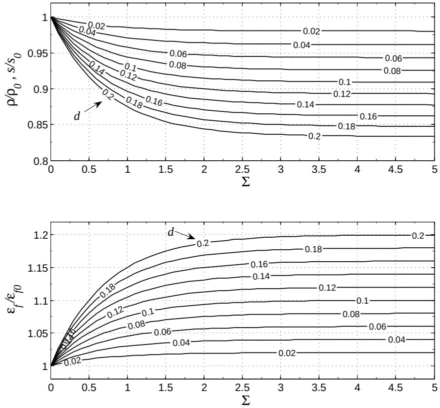
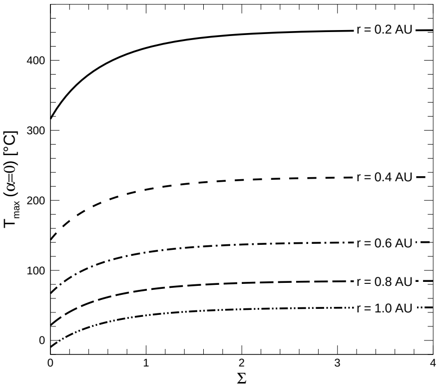
OSSD Effects

on the optical coefficients and the maximum sail temperature



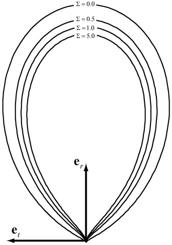
ρ/ρ_0, s/s_0, and $\varepsilon_f/\varepsilon_{f0}$



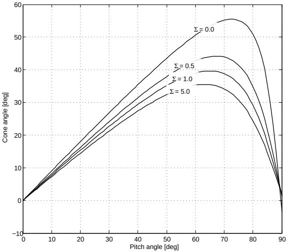
T_{max} for different solar distances ($d = 0.2$)

OSSD Effects

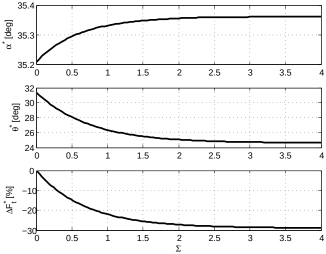
on the SRP force bubble and the control angles



$\mathbf{F}_{\mathrm{SRP}}$-bubble



$\theta(\alpha)$



α^*, θ^*, and ΔF_t^*

Simulation Model

Considerations for trajectory control

- Gravitational forces of all celestial bodies
- Solar wind
- Finiteness of the solar disk
- Reflected light from close celestial bodies
- Aberration of solar radiation (Poynting-Robertson effect)
- The solar sail bends and wrinkles, depending on the actual solar sail design
- Finite attitude control maneuvers

Simplifications for mission feasibility analysis and to isolate the effects of OSSD

- The solar sail is a flat plate
- The solar sail is moving under the sole influence of solar gravitation and radiation
- The sun is a point mass and a point light source
- The solar sail attitude can be changed instantaneously

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Problem Statement

Equations of motion

For a solar sail in heliocentric cartesian reference frame:

$$\dot{\mathbf{r}} = \mathbf{v}$$
$$\dot{\mathbf{v}} = -\frac{\mu}{r^3}\mathbf{r} + \mathbf{a}$$

where

$$\mathbf{a} = \mathbf{a}(r, \mathbf{n}, b_1(t), b_2(t), b_3(t))$$

is the SRP acceleration acting on the solar sail

Problem

Minimize the time t_f necessary to transfer the sail from $\mathbf{x}_0 = (\mathbf{r}_0, \mathbf{v}_0)$ to $\mathbf{x}_f = (\mathbf{r}_f, \mathbf{v}_f)$ by maximizing the performance index $J = -t_f$

Nomenclature

$\mathbf{a}$: propulsive (+ disturbing) acceleration on the sail

$\mathbf{r}$: sail position

r: radius, $|\mathbf{r}|$

$\mathbf{v}$: sail velocity

μ: gravitational parameter of the sun

$b_1(t)$, $b_2(t)$, $b_3(t)$: functions of the sail's optical parameters

$\mathbf{n}$: sail normal vector

$\mathbf{x}$: sail state

$\square_0$: initial value of $\square$

$\square_f$: final value of $\square$

Problem Statement

Equations of motion

For a solar sail in heliocentric cartesian reference frame:

$$\dot{\mathbf{r}} = \mathbf{v}$$

$$\dot{\mathbf{v}} = -\frac{\mu}{r^3}\mathbf{r} + \mathbf{a}$$

where

$$\mathbf{a} = \mathbf{a}(r, \mathbf{n}, b_1(t), b_2(t), b_3(t))$$

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Variational Problem

Using standard COV, the optimal direction of **n** is found by maximizing the Hamiltonian $\mathcal{H}$

$$\mathcal{H} = \boldsymbol{\lambda}_r \cdot \mathbf{v} - \frac{\mu}{r^3}\boldsymbol{\lambda}_v \cdot \mathbf{r} + \boldsymbol{\lambda}_v \cdot \mathbf{a} + \lambda_\Sigma \frac{r_0^2}{r^2 T}\mathbf{e}_r \cdot \mathbf{n}$$

where $\boldsymbol{\lambda}_r$, $\boldsymbol{\lambda}_v$ are the vectors adjoint to the position, and λ_Σ is the radiation dose costate

The result is

$$\mathbf{n} = \begin{cases} \dfrac{\sin(\alpha_\lambda - \alpha)}{\sin\alpha_\lambda}\mathbf{e}_r + \dfrac{\sin\alpha}{\sin\alpha_\lambda}\mathbf{e}_{\boldsymbol{\lambda}_v} & \text{for} \quad \alpha_\lambda \neq 0 \\ \mathbf{e}_r & \text{for} \quad \alpha_\lambda = 0 \end{cases}$$

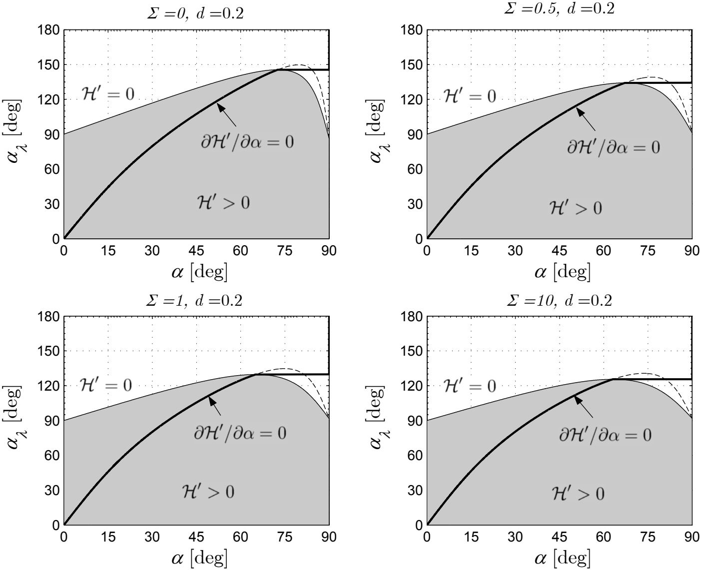
where

$$\mathbf{e}_r = \frac{\mathbf{r}}{|\mathbf{r}|} \qquad \mathbf{e}_{\boldsymbol{\lambda}_v} = \frac{\boldsymbol{\lambda}_v}{|\boldsymbol{\lambda}_v|} \qquad \cos\alpha_\lambda = \mathbf{e}_r \cdot \mathbf{e}_{\boldsymbol{\lambda}_v}$$

Remarks about the Optimal Solution

- The optimal control law requires the thrust vector to lie in the plane defined by the position vector **r** and the primer vector $\boldsymbol{\lambda}_v$. This generalizes a similar conclusion obtained for model IR by C. Sauer and for model NPR without degradation by G. Mengali and A. Quarta
- The equation giving the optimal cone angle as a function of α_λ can be written analytically and solved numerically
- The next slide shows, how the optimal solutions typically vary with the solar radiation dose

Typical Optimal Solutions

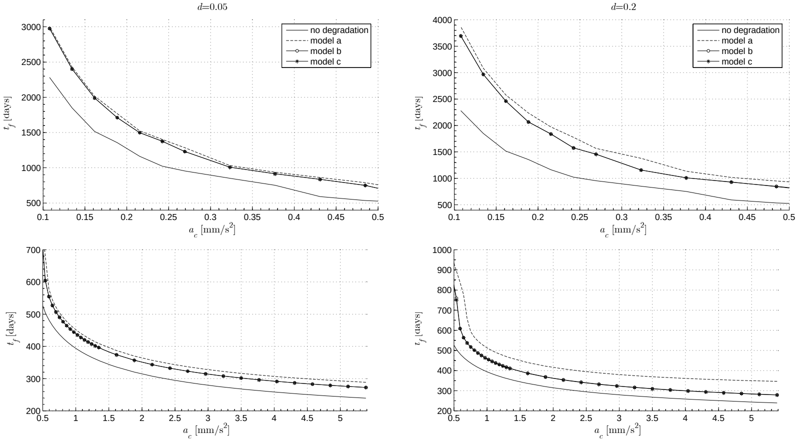


Mars Rendezvous

- Solar sail with $0.1\,\mathrm{mm/s^2} \leq a_c < 6\,\mathrm{mm/s^2}$
- $C_3 = 0\,\mathrm{km^2/s^2}$
- 2D-transfer from circular orbit to circular orbit
- Trajectories calculated by G. Mengali and A. Quarta using a classical indirect method with an hybrid technique (genetic + gradient-based algorithm) to solve the associated boundary value problem
- Degradation factor: $0 \leq d \leq 0.2$ (0–20% degradation limit)
- Half life SRD: $\hat{\Sigma} = 0.5\,(S_0 \cdot \mathrm{yr})$
- Three models:
  - ▷ Model (a): Instantaneous degradation
  - ▷ Model (b): Control neglects degradation ("ideal" control law)
  - ▷ Model (c): Control considers degradation

Mars Rendezvous

Trip times for 5% and 20% degradation limit



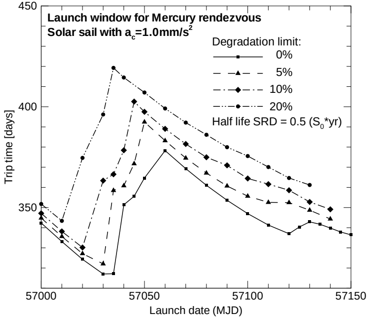
- OSSD has considerable effect on trip times
- The results for model (b) and (c) are indistinguishable close

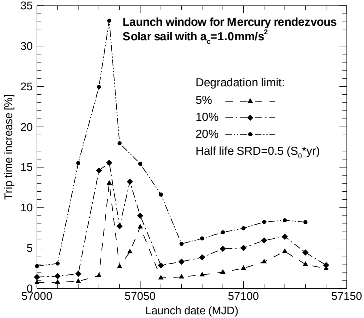
Mercury Rendezvous

- Solar sail with $a_c = 1.0\,\mathrm{mm/s^2}$
- $C_3 = 0\,\mathrm{km^2/s^2}$
- Trajectories calculated by B. Dachwald with the trajectory optimizer GESOP with SNOPT
- Arbitrarily selected launch window $\mathrm{MJD}\,57000 \leq t_0 \leq \mathrm{MJD}\,57130$ (09 Dec 2014 – 18 Apr 2015)
- Final accuracy limit was set to $\Delta r_{f,\max} = 80\,000\,\mathrm{km}$ (inside Mercury's sphere of influence at perihelion) and $\Delta v_{f,\max} = 50\,\mathrm{m/s}$
- Degradation factor: $0 \leq d \leq 0.2$ (0–20% degradation limit)
- Half life SRD: $\hat{\Sigma} = 0.5\,(S_0 \cdot \mathrm{yr})$

Mercury Rendezvous

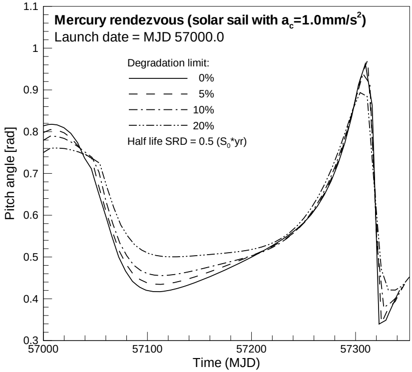
Launch window for different *d*



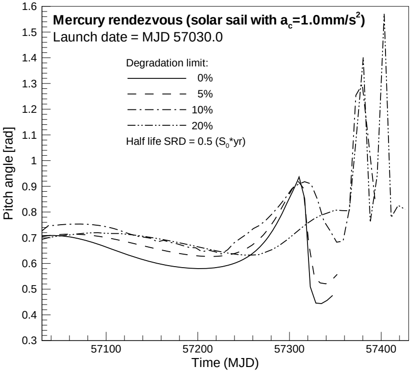


- Sensitivity of the trip time with respect to OSSD depends considerably on the launch date
- Some launch dates considered previously as optimal become very unsuitable when OSSD is taken into account
- For many launch dates OSSD does not seriously affect the mission

Mercury Rendezvous
Optimal α-variation for different *d*



Launch at MJD 57000.0



Launch at MJD 57030.0

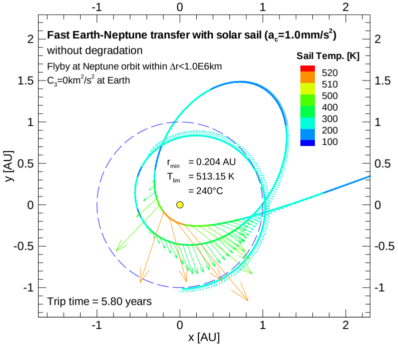
- OSSD can also have remarkable consequences on the optimal control angles
- Given an indefinite OSSD behavior at launch, MJD 57000.0 would be a very robust launch date

Fast Neptune Flyby

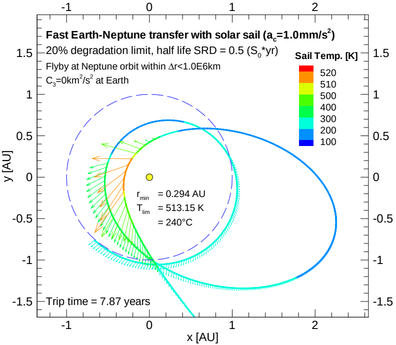
- Solar sail with $a_c = 1.0\,\mathrm{mm/s^2}$
- $C_3 = 0\,\mathrm{km^2/s^2}$
- Trajectories calculated by B. Dachwald with the trajectory optimizer InTrance
- To find the absolute trip time minima, independent of the actual constellation of Earth and Neptune, no flyby at Neptune itself, but only a crossing of its orbit within a distance $\Delta r_{f,\max} < 10^6$ km was required, and the optimizer was allowed to vary the launch date within a one year interval
- Sail film temperature was limited to 240°C by limiting the sail pitch angle
- Degradation factor: $0 \le d \le 0.2$ (0–20% degradation limit)
- Half life SRD: $0 \le \hat{\Sigma} \le 2$ $(S_0 \cdot \mathrm{yr})$

Fast Neptune Flyby

Topology of optimal trajectories for different *d*



$d = 0$



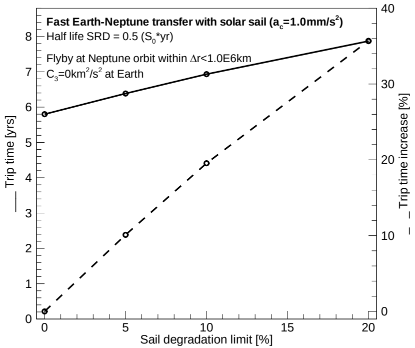
$d = 0.2$

With increasing degradation:

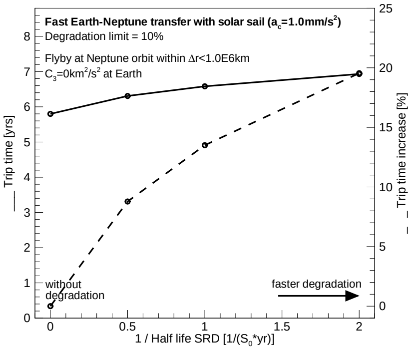
- Increasing solar distance during final close solar pass
- Increasing solar distance before final close solar pass
- Longer trip time

Fast Neptune Flyby

Trip time and trip time increase for different d and $\hat{\Sigma}$



Different degradation factors d ($\hat{\Sigma} = 0.5\,(S_0 \cdot \mathrm{yr})$)



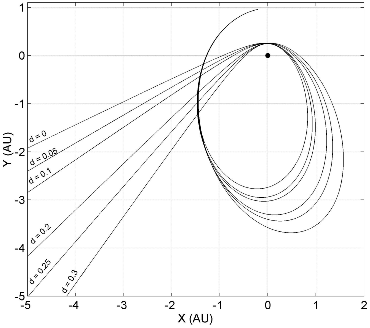
Different half life SRDs $\hat{\Sigma}$ ($d = 0.1$)

Fast Transfer to the Heliopause

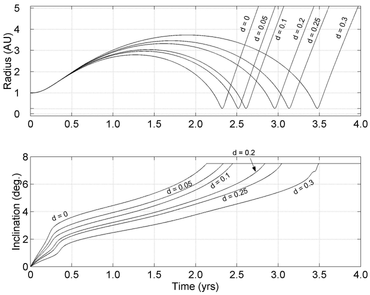
- Solar sail with $a_c = 1.75\,\mathrm{mm/s^2}$
- $C_3 = 0\,\mathrm{km^2/s^2}$
- Trajectories calculated by M. Macdonald with A^nD-blending (blending of locally optimal control laws)
- Transfer to the nose of the heliosphere at a latitude of 7.5 deg and a longitude of 254.5 deg at 200 AU from the sun
- Sail jettison at 5 AU to eliminate any potential interference with the interplanetary/interstellar medium
- Solar distance limited to 0.25 AU
- Degradation factor: $0 \leq d \leq 0.3$ (0–30% degradation limit)
- Half life SRD: $\hat{\Sigma} = 0.5\,(S_0 \cdot \mathrm{yr})$

Fast Transfer to the Heliopause

Trajectories for different d ($\hat{\Sigma} = 0.5\,(S_0 \cdot \text{yr})$)



Inner solar system trajectories



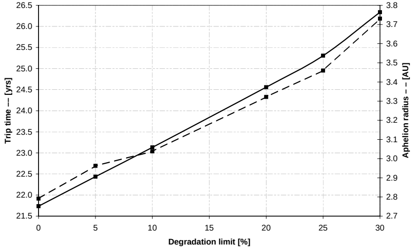
Variation of solar distance and inclination

With increasing degradation:

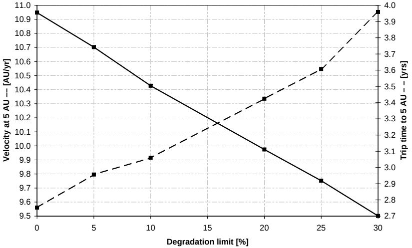
- Constant solar distance during final close solar pass
- Increasing radius of aphelion passage

Fast Transfer to the Heliopause

Trajectories for different d ($\hat{\Sigma} = 0.5\,(S_0 \cdot \mathrm{yr})$)



Trip time to 200 AU and radius of aphelion passage



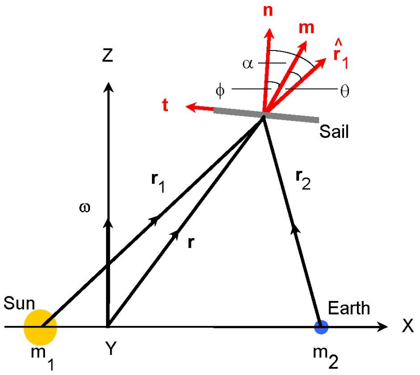
Trip time and velocity at 5 AU (sail jettison point)

With increasing degradation:

- Increasing radius of aphelion passage
- Longer trip time

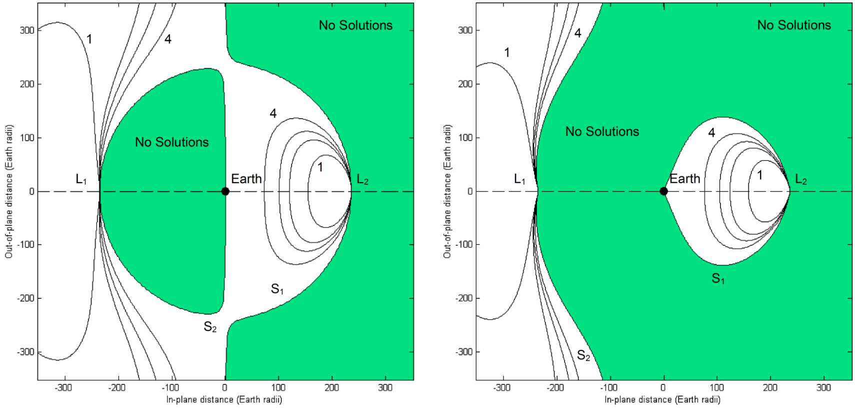
Artificial Lagrange-Point Missions

- Sun-Earth restricted circular three-body problem with non-perfectly solar sail
- SRP acceleration allows to hover along artificial equilibrium surfaces (manifold of artificial Lagrange-points)
- Solutions calculated by C. McInnes



Artificial Lagrange-Point Missions

Contours of sail loading in the *x*-*z*-plane

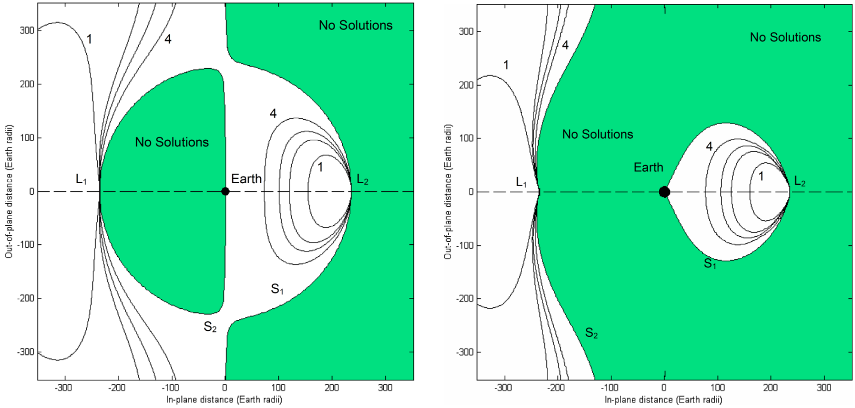


$\rho = 1$ $\rho = 0.9$

[1] $30\,\mathrm{g/m^2}$ [2] $15\,\mathrm{g/m^2}$ [3] $10\,\mathrm{g/m^2}$ [4] $5\,\mathrm{g/m^2}$

Artificial Lagrange-Point Missions

Contours of sail loading in the *x*-*z*-plane

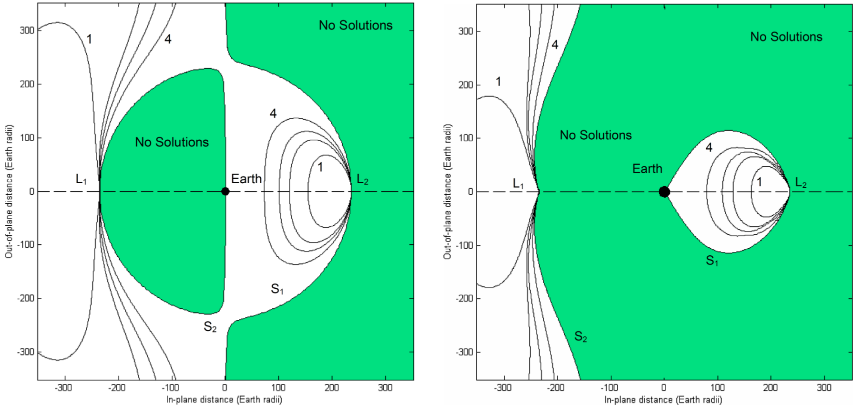


$\rho = 1$ $\qquad$ $\rho = 0.8$

[1] $30\,\mathrm{g/m^2}$ [2] $15\,\mathrm{g/m^2}$ [3] $10\,\mathrm{g/m^2}$ [4] $5\,\mathrm{g/m^2}$

Artificial Lagrange-Point Missions

Contours of sail loading in the *x*-*z*-plane



$\rho = 1$ $\rho = 0.7$

[1] $30\,\mathrm{g/m^2}$ [2] $15\,\mathrm{g/m^2}$ [3] $10\,\mathrm{g/m^2}$ [4] $5\,\mathrm{g/m^2}$

Summary and Conclusions

- Based on the current standard model for non-perfectly reflecting solar sails, we have developed a parametric model that includes the optical degradation of the sail film due to the erosive effects of the space environment
- Using this model, we have investigated the effect of different potential degradation behaviors on trajectory and attitude control for various exemplary missions
- All our results show that, in general, optical solar sail degradation has a considerable effect on trip times and on the optimal steering profile. For specific launch dates, especially those that are optimal without degradation, this effect can be tremendous

Outlook

- Having demonstrated the *potential* effects of optical solar sail degradation on future missions, more research on the *real* degradation behavior has to be done because the degradation behavior of solar sails in the real space environment is to a considerable degree indefinite
- To narrow down the ranges of the parameters of our model, further laboratory tests have to be performed
- Additionally, before a mission that relies on solar sail propulsion is flown, the candidate solar sail films have to be tested in the relevant space environment
- Some near-term missions currently studied in the US and Europe would be an ideal opportunity for testing and refining our degradation model

Potential Solar Sail Degradation Effects on Trajectory and Attitude Control

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