Solar Sail Trajectory Optimization for the Solar Polar Imager (SPI) Mission

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Outline

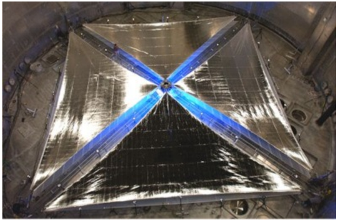
The Solar Polar Imager Mission

- SPI mission is one of several Sun-Earth Connection solar sail roadmap missions currently envisioned by NASA
- Objectives:
  - To investigate the global structure and dynamics of the solar corona
  - To reveal the secrets of the solar cycle and the origins of solar activity
- Target orbit is a heliocentric circular orbit at 0.48 AU with an inclination of 75 deg
  - 3:1 resonance with Earth
  - different target inclinations have been considered in various previous studies
- Similar solar sail mission, called Solar Polar Orbiter (SPO), is studied by ESA

Solar Sailcraft Design for the SPI Mission

- 160 m $\times$ 160 m, 150 kg square solar sail assembly
- 250 kg spacecraft bus
- 50 kg scientific payload
- 450 kg total mass
- Characteristic thrust (max. thrust at 1 AU): $F_c = 160\,\mathrm{mN}$
- Characteristic acceleration (max. acceleration at 1 AU): $a_c = 0.35\,\mathrm{mm/s^2}$

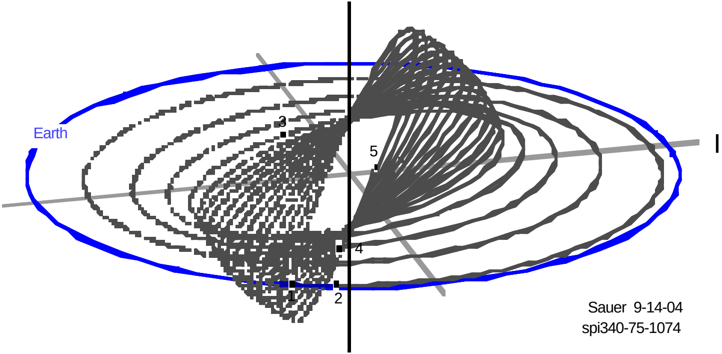
DLR solar sail deployment test 1999©ESA



ATK solar sail deployment test 2005©NASA

Reference Solution

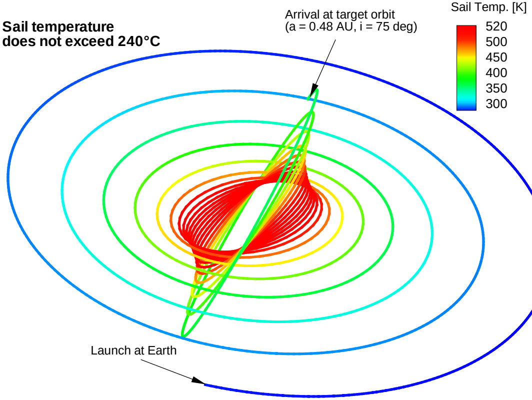
- Sail film temperature: $T < 100°\mathrm{C}$
- Hyperbolic excess energy: $C_3 = 0.25\,\mathrm{km}^2/\mathrm{s}^2$



Mission duration: 6.7 years

Preview of Our Hot Solution

- Sail film temperature limit: $T_{\text{lim}} = 240^{\circ}\text{C}$
- Hyperbolic excess energy: $C_3 = 0\,\text{km}^2/\text{s}^2$



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The Non-Perfectly Reflecting Solar Sail

The non-perfectly reflecting solar sail model parameterizes the optical behavior of the sail film by the optical coefficient set

$$\mathcal{P} = \{\rho, s, \varepsilon_\mathrm{f}, \varepsilon_\mathrm{b}, B_\mathrm{f}, B_\mathrm{b}\}$$

The optical coefficients for a solar sail with a highly reflective aluminum-coated front side and with a highly emissive chromium-coated back side are:

$$\mathcal{P}_{\mathrm{Al|Cr}} = \{\rho = 0.88, s = 0.94, \varepsilon_\mathrm{f} = 0.05, \varepsilon_\mathrm{b} = 0.55, B_\mathrm{f} = 0.79, B_\mathrm{b} = 0.55\}$$

ρ: reflection coefficient

s: specular reflection factor

ε_f and ε_b: emission coefficients of the front and back side, respectively

B_f and B_b: non-Lambertian coefficients of the front and back side, respectively

Simulation Model

Considerations for high-precision trajectory control:

- Solar sail bends and wrinkles, depending on actual solar sail design
- Gravitational forces of all celestial bodies
- Reflected light from close celestial bodies
- Solar wind
- Finiteness of solar disk
- Finite low-precision attitude control maneuvers
- Aberration of solar radiation (Poynting-Robertson effect)

Allowed simplifications for mission feasibility analysis:

- Solar sail is a flat plate
- Solar sail is moving under sole influence of solar gravitation and radiation
- Sun is a point mass and a point light source
- Solar sail attitude can be changed instantaneously

Local Steering Laws (LSLs)

- LSLs give locally optimal thrust direction to change some specific osculating orbital element of spacecraft with maximum rate
- In an orbital reference frame $\mathcal{O} = \{\mathbf{e}_r, \mathbf{e}_t, \mathbf{e}_h\}$, the equations for the semi-major axis a and the inclination i can be written as

$$\frac{da}{dt} = \frac{2a^2}{h}\left(e \sin f \, a_r + \frac{p}{r} a_t\right)$$
$$\frac{di}{dt} = \frac{r\cos(\omega + f)}{h} a_h$$

a_r, a_t, a_h: acceleration components along the radial, transversal, and orbit normal direction

e: eccentricity

f: true anomaly

h: orbital angular momentum per spacecraft unit mass

p: semilatus rectum

r: radius

ω: argument of perihelion

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$$\frac{da}{dt} = \begin{pmatrix} \frac{2a^2}{h} e \sin f \\ \frac{2a^2}{h}\frac{p}{r} \\ 0 \end{pmatrix} \cdot \begin{pmatrix} a_r \\ a_t \\ a_h \end{pmatrix} = \mathbf{k}_a \cdot \mathbf{a}$$

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$\mathbf{a}$: acceleration vector

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How to Determine the Optimal Thrust Direction

$$\frac{da}{dt} = \begin{pmatrix} \frac{2a^2}{h} e \sin f \\ \frac{2a^2}{h}\frac{p}{r} \\ 0 \end{pmatrix} \cdot \begin{pmatrix} a_r \\ a_t \\ a_h \end{pmatrix} = \mathbf{k}_a \cdot \mathbf{a}$$

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Now it is clear that to decrease the semi-major axis with a maximum rate, the thrust vector has to be along the direction $-\mathbf{k}_a$ (local steering law $\mathcal{L}_{a-}$). To increase the inclination with a maximum rate, the thrust vector has to be along the direction $\mathbf{k}_i$ (local steering law $\mathcal{L}_{i+}$)

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Evolutionary Neurocontrol (ENC)

A smart global trajectory optimization method

- We used ENC to calculate near-globally optimal trajectories
- ENC is based on a combination of artificial neural networks with evolutionary algorithms
- ENC attacks trajectory optimization problems from the perspective of artificial intelligence and machine learning
- ENC was implemented within a low-thrust trajectory optimization program called InTrance (**In**telligent **Tra**jectory optimization using **n**euro**c**ontroller **e**volution)
- InTrance requires only the target body/state and intervals for the initial conditions as input to find a near-globally optimal trajectory for the specified problem
- InTrance works without an initial guess and does not require the attendance of a trajectory optimization expert

Determination of the SPI Trajectory Using Local Steering Laws

Using LSLs, the strategy to attain the SPI target orbit divides the trajectory into the following phases:

1: Spiralling inwards until the SPI target semi-major axis is reached (using local steering law $\mathcal{L}_{a-}$)

2: Cranking the orbit until the SPI target inclination is reached (using local steering law $\mathcal{L}_{i+}$)

3: Circularizing the orbit until the SPI target orbit is attained (using a combination of the local steering laws $\mathcal{L}_{a-}$, $\mathcal{L}_{a+}$, and $\mathcal{L}_{e-}$)

Determination of the Optimal Semi-Major Axis for Orbit Cranking

- Acceleration of solar sails is proportional to $1/r^2$
- Minimum solar distance is constrained by the temperature limit of the sail film T_{lim}
- Equilibrium temperature of the sail film is

$$T \propto \left(\frac{r_0}{r}\right)^{1/2} \cos^{1/4} \alpha$$

r: solar distance

r_0: 1 AU

α: sail pitch angle

- T_{lim} can be used directly as optimization constraint by constraining the pitch angle $\alpha > \alpha_{\text{lim}}(r, T_{\text{lim}})$
- The time required to crank the orbit depends on the orbit cranking semi-major axis a_{cr}

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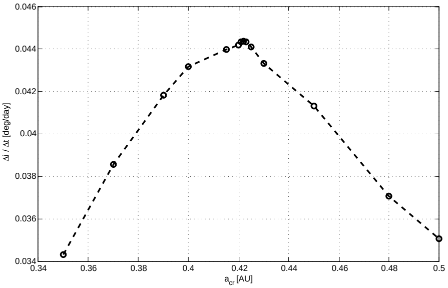
An optimal orbit cranking semi-major axis $a_{\text{cr,opt}}(T_{\text{lim}})$ exists, where the inclination change rate $\frac{\Delta i}{\Delta t}$ is maximal

- $a_{\text{cr}} > a_{\text{cr,opt}} \longrightarrow$ lower SRP
- $a_{\text{cr}} < a_{\text{cr,opt}} \longrightarrow$ ineffectively large α_{lim} to keep $T < T_{\text{lim}}$



$a_{\text{cr,opt}}(T_{\text{lim}} = 100°\text{C}) = 0.422\,\text{AU} \longrightarrow \frac{\Delta i}{\Delta t} = 0.0444\,\text{deg/day}$

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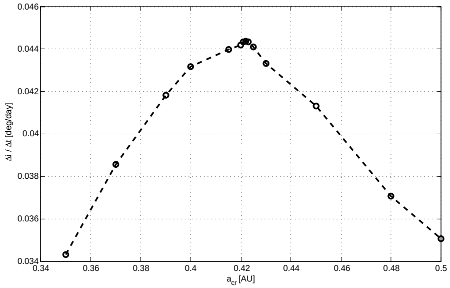

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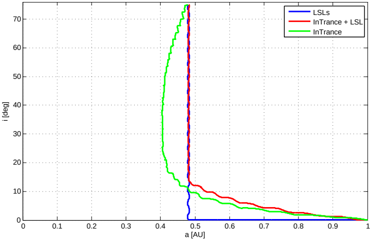
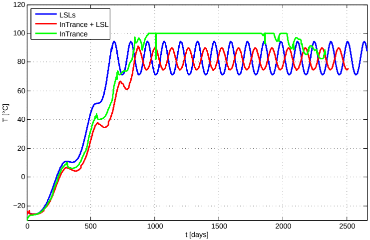
Comparison of Different Solutions



Inclination over semi-major axis



Sail film temperature over flight time

Method	Δt [years]	$T_{\max}$ [$^\circ$C]
LSLs	7.28	95
InTrance + LSL	6.88	91
InTrance	6.39	100

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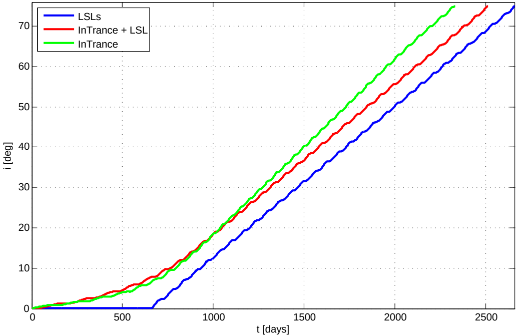

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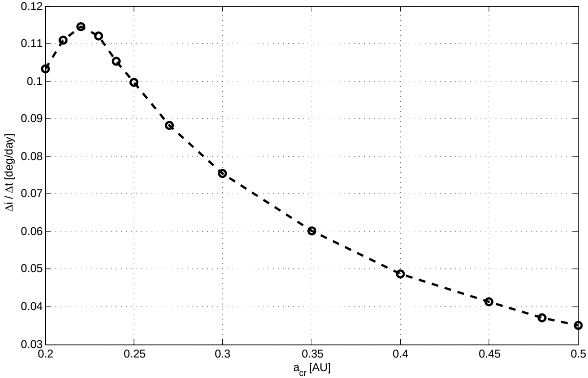
2: Cranking the orbit until the SPI target inclination is reached (using local steering law $\mathcal{L}_{i^+}$)

3: Spiralling outwards until the SPI target semi-major is reached (using local steering law $\mathcal{L}_{a^+}$)

4: Circularizing the orbit until the SPI target orbit is attained (using a combination of the local steering laws $\mathcal{L}_{a^-}$, $\mathcal{L}_{a^+}$ and $\mathcal{L}_{e^-}$)

Optimal Semi-Major Axis for Orbit Cranking

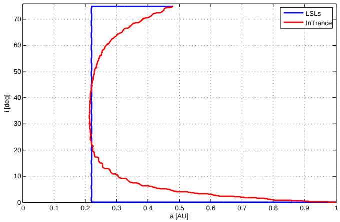


$$a_{\mathrm{cr,opt}}(T_{\mathrm{lim}} = 240^\circ\mathrm{C}) = 0.22\,\mathrm{AU} \rightarrow \frac{\Delta i}{\Delta t} = 0.1145\,\mathrm{deg/day}$$

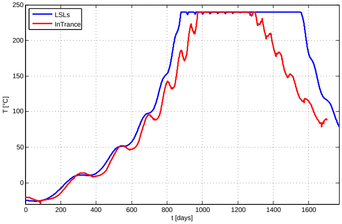
Remember:

$$a_{\mathrm{cr,opt}}(T_{\mathrm{lim}} = 100^\circ\mathrm{C}) = 0.422\,\mathrm{AU} \rightarrow \frac{\Delta i}{\Delta t} = 0.0444\,\mathrm{deg/day}$$

Comparison of the LSL-Solution with the InTrance-Solution



Inclination over semi-major axis

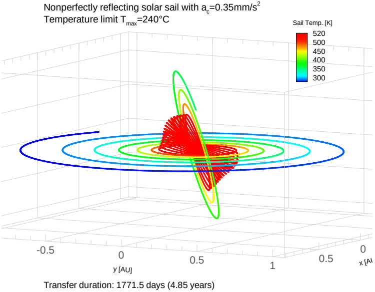


Sail film temperature over flight time

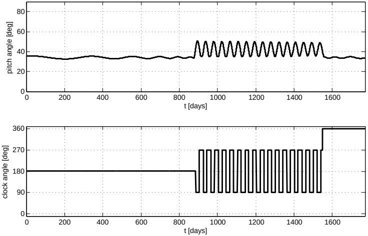
Method	Δt [years]	$T_{\max}$ [°C]
LSLs	4.85	240
InTrance	4.66	240

Method	Δt [years]	$T_{\max}$ [°C]
InTrance+LSLs	6.88	91
InTrance	6.39	100

LSL-Solution



3D trajectory plot



Solar sail control angles

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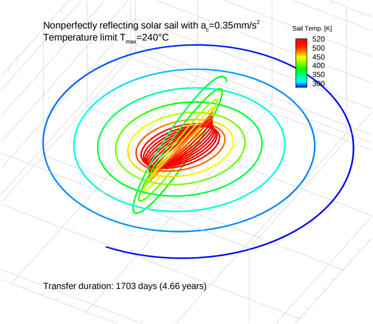
InTrance-Solution



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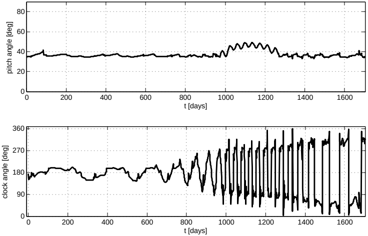
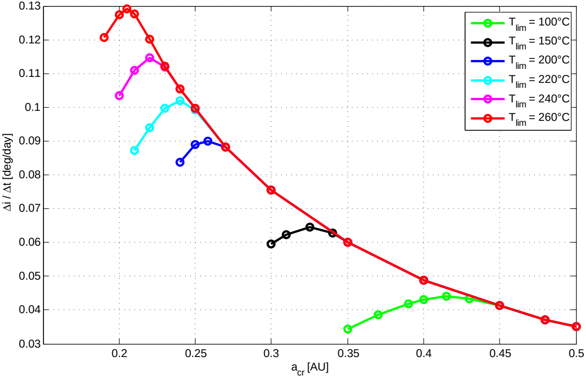


Solar sail control angles

Variation of the Sail Temperature Limit



- The optimal orbit-cranking semi-major axis can be approximated with an error of less than 2% by

$$\tilde{a}_{cr,opt} \approx 1.4805 - 0.23 \cdot \ln(\tilde{T}_{lim})$$

- The maximum inclination change rate can be approximated with an error of less than 2% by

$$(\widetilde{\Delta i/\Delta t})_{max} \approx 0.0113 \cdot \tilde{a}_{cr,opt}^{-1.53}$$

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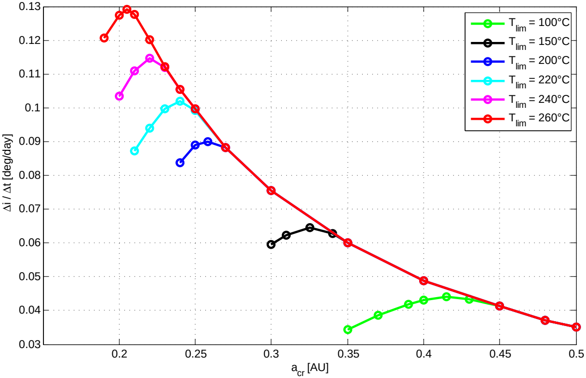

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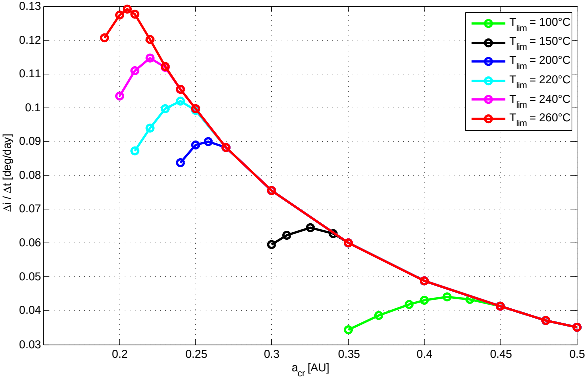

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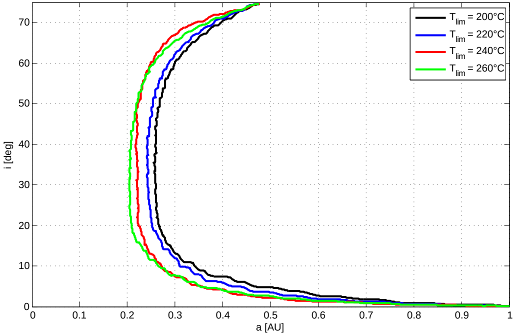
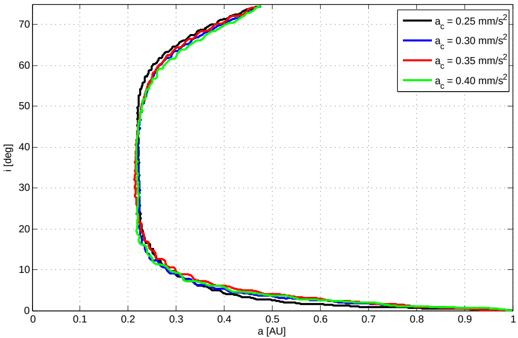


Inclination over semi-major axis, $i(a)$

T_{lim} [°C]	$a_{\mathrm{cr,opt}}$ [AU]	$(\Delta i/\Delta t)_{\mathrm{max}}$ [deg/day]	Δt [years]
200	0.260	0.0899	4.90
220	0.236	0.1015	4.75
240	0.220	0.1145	4.60
260	0.205	0.1291	4.50

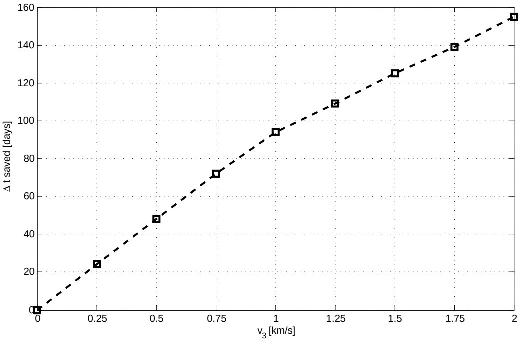
Variation of the Characteristic Acceleration



Inclination over semi-major axis, $i(a)$

a_c $[\mathrm{mm/s^2}]$	Δt [years]
0.25	6.48
0.3	5.39
0.35	4.60
0.4	4.10

Variation of the Hyperbolic Excess Energy (C_3) / Velocity ($v_3 = \sqrt{C_3}$)



Transfer time saved by injection with some hyperbolic excess velocity

Thus a C_3 of $0.25\,\mathrm{km}^2/\mathrm{s}^2$ makes the reference trajectory about 50 days (0.13 years) faster w.r.t. to the C_3 of $0\,\mathrm{km}^2/\mathrm{s}^2$ that is used in our mission design

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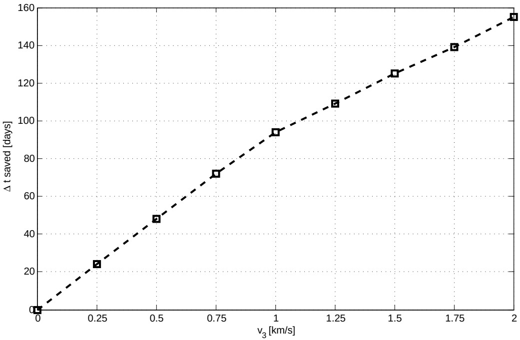

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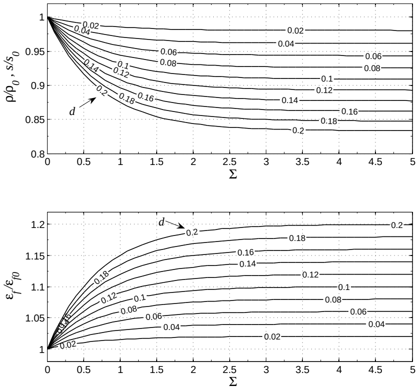
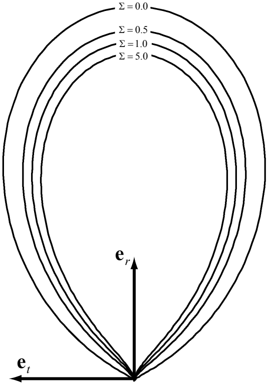
Solar Sail Degradation Model

(by Dachwald et al.)



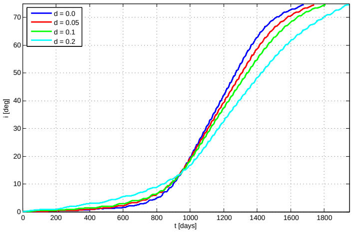
"degradation" of optical coefficients



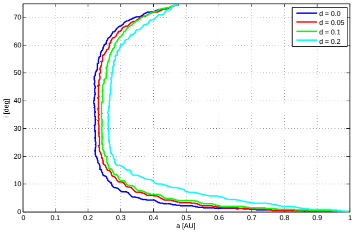
"degradation" of SRP force bubble

Solar Sail Degradation

"Half life" solar radiation dose $\hat{\Sigma} = 25\, S_0 \cdot \mathrm{yr} = 394\, \mathrm{TJ/m^2}$



Inclination over flight time



Inclination over semi-major axis

Degradation factor	Δt [years]
0.0	4.60
0.05	4.77
0.1	4.96
0.2	5.33

- $\Delta i / \Delta t$ becomes smaller with increasing SRD
- For larger degradation factors it is favorable to crank the orbit further away from the sun

Summary and Conclusions

- A current SPI reference mission design employs a cold mission scenario, where the sail film temperature stays colder than 100°C (a quite conservative value). It spirals inwards to 0.48 AU and then cranks the orbit to 75 deg (transfer time is 6.9 years for $C_3 = 0\,\mathrm{km}^2/\mathrm{s}^2$)
- Using this temperature limit as a direct constraint (instead of a solar distance limit), we have found a faster transfer trajectory (6.4 years) that approaches the sun to about 0.4 AU solar distance and thus better exploits the solar radiation pressure
- For hot mission scenarios (higher sail temperature limits of 200-260°C), the optimal transfer trajectories approach the sun even closer (to about 0.20-0.26 AU solar distance), resulting in even shorter transfer durations (4.5-4.9 years)
- We have also performed various tradeoffs for the hot mission scenario to gain a deeper insight into the trade space of the SPI mission and to help the designer of such a mission to estimate the required transfer time

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- For hot mission scenarios (higher sail temperature limits of 200-260°C), the optimal transfer trajectories approach the sun even closer (to about 0.20-0.26 AU solar distance), resulting in even shorter transfer durations (4.5-4.9 years)
- We have also performed various tradeoffs for the hot mission scenario to gain a deeper insight into the trade space of the SPI mission and to help the designer of such a mission to estimate the required transfer time

Summary and Conclusions

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SPI Mission Challenges

- The mission performance might be seriously affected by optical degradation of the sail surface, as it is expected in the extreme space environment close to the sun. Due to the unknown degradation behavior of solar sails in the space environment, ground and in-space tests are required
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Solar Sail Trajectory Optimization for the Solar Polar Imager (SPI) Mission

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